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Assessment of interspecies scattering lengths a_{12} from stability of two-component Bose-Einstein condensates

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Abstract. A stability method is used to assess possible values of interspecies scattering lengths a_{12} in two-component Bose-Einstein condensates described within the Gross-Pitaevskii approximation. The technique, based on a recent stability analysis of solitonic excitations in two-component Bose-Einstein condensates, is applied to ninety combinations of atomic alkali pairs with given singlet and triplet intraspecies scattering lengths as input parameters. Results obtained for values of a_{12} are in a reasonable agreement with the few ones available in the literature and with those obtained from a Painlevé analysis of the coupled Gross-Pitaevskii equations.

PACS. 03.75.Lm Tunnelling, Josephson effect, Bose-Einstein condensates in periodic potentials, solitons, vortices and topological excitations – 03.75.Mn Multicomponent condensates; spinor condensates – 03.75.Nt Other Bose-Einstein condensation phenomena – 42.65.Tg Optical solitons; nonlinear guided waves – 42.81.Dp Propagation, scattering, and losses; solitons

1 Introduction

Over the past decade an intensive interest is devoted to atomic and molecule Bose-Einstein condensation. Recent progress in experimental techniques makes it possible in the nearest future to engineer two-component Bose-Einstein condensates (BECs) that are composed of two different alkali-metal atoms. Until now, merely theoretical studies have been carried out for two-component alkali-metal atom BEC systems like Li-Rb [1], Na-Rb [1–4], and K-Rb [5,6]. However, the mixture Cs-Li [7,8] has been investigated experimentally, without reaching the BEC phase.

One of the most important quantities in engineering BECs is the scattering length a_{ij} characterizing the strength of the interaction between the atoms of types i and j. A realization of two-component BECs depends on three scattering lengths, namely the intraspecies scattering lengths a_{11}, a_{22} and the interspecies ones $a_{12} = a_{21}$. Accurate knowledge of these three quantities is of great importance in design and implementation of two-component BECs. However, the problem is that the scattering lengths between many like alkali-metal atoms are known, whereas those between unlike alkalis have not yet been measured or calculated, except for very few cases (e.g. [5–7, 9–11]).

The aim of this note is to apply a simple stability method to assess possible values of interspecies scattering length by using a recent analysis [1] for existence of solitons within the two-component BEC. The basic idea of the formulation is that the solitons are stable objects and if they can be created in the background BEC matter then the latter should show up also stability property. Various types of solitonic excitations can be created in two-component BEC matter, such as those of bright (B) and dark (D) type or of BB and DD types. Solitonic excitations in BEC matter is dealt with by many authors including [12–17]. In this work we shall treat only BD type excitations which provide finite interval for possible values of interspecies scattering length a_{12}.

We shall determine, in tabulated form, those ranges of a_{12} for which various two-component BECs maintain stability with respect to the solitonic excitations. The background is treated in the Thomas-Fermi (TF) approximation while the excitations are considered in the form of static BD solitons. Furthermore, we restrict the treatment of the dynamics to one dimension, but employ three-dimensional scattering lengths using an appropriate crossing area A. Note that cigar-like quasi one-dimensional BECs have already been produced by micro-trap devices developed quite recently [18–21]. As a case study we shall treat in detail two specific BEC systems, namely those composed of the alkali-metal atom pairs K-Rb and Li-Na. For these systems we exhibit the BD solitonic density profiles at the most probable values of a_{12} which ensure integrability (i.e. inverse scattering solution of the Gross-Pitaevksii equations), as well as the allowable range of a_{12} which takes care of particle number conservation.

The results of the present investigation indicate that given the intraspecies scattering length a_{ii}, $i = 1, 2$ the method is capable to provide possible values for the interspecies scattering length a_{12}, at which the two-component

BEC may prove stable. Whether or not the true (physical) values of a_{12} fall into these stability ranges is a question. If yes, then one can further calculate for finer details of the two-component BEC, as for the particle numbers N_i $(i = 1, 2)$, the extension of components and so on. If not, then it would mean that other types (e.g. BB or DD) of solitonic excitations compatible with the physical value of a_{12} could be realized. Moreover, one may try to adjust a_{ij}'s slightly by changing the strength of the magnetic field [22], to meet the stability requirement formulated in the present work. (Note, however, that the Feshbach resonance method is not fully developed for interatomic collisions.) A third possibility is to choose other alkali pair being more favorable from point of view of values of a_{ij}'s for assembling the two-component BEC satisfying stability condition formulas given in the present method.

In section 2 the theoretical background will be outlined. Section 3 contains the estimated ranges of interspecies scattering length a_{12}, in tabulated form, for which the solitonic stability requirement holds and a comparison with earlier findings and those resulted from Painlevé analysis will be made. Section 4 is devoted to a short summary of the method.

2 Theoretical background

Following [1] we write the coupled Gross-Pitaevskii (GP) equations [23,24] describing a quasi one-dimensional two-component BEC in the form $(i = 1, 2)$:

$$i\hbar\psi_{i,t} = \left[-\frac{\hbar^2}{2m_i}\,\partial_{xx} + \sum_{j=1}^{2}\Omega_{ij}\,|\,\psi_j|^2 + V_i\right]\psi_i \qquad (1)$$

where m_i denotes the individual mass of the ith atomic species, $\Omega_{ij} = 2\pi\hbar^2 a_{ij}/A\mu_{ij}$ with a_{ij} being the 3D scattering length, A represents a general transverse crossing area of the cigar-shape BEC, $\mu_{ij} = m_i m_j/(m_i + m_j)$ is the reduced mass, and V_i, $(i = 1, 2)$ denotes the external trapping potentials. In the case of real trap potentials the normalization of the wave functions reads as $N_i = \int_{-\infty}^{\infty}|\,\psi_i|^2 dx$, $(i = 1, 2)$ with N_i denoting the number of atoms in the ith component. We do not allow for the species to transform into each other, so that the numbers of particles are conserved quantities for both alkalies.

The stationary solutions of the coupled GP equation (1) can be written in the form $\psi_i(x,t) = \Phi_i(x)\exp(-iE_i t/\hbar)$ where E_i represents the one-particle energy of the $i-$th component particle $(i = 1, 2)$. Using this formula and neglecting the kinetic terms one is able to derive the approximate density profiles and a semi-infinite range for a_{12} in which the Thomas-Fermi ground state exists. In the case of a harmonic trap potential the calculation results in the usual inverse-parabolic distribution.

We then look for the excited static solutions in the form $\tilde{\psi}_i(x,t) = \Phi_i(x)\phi_i(x)\exp(-i\tilde{E}_i t/\hbar)$ with $\phi_i(x)$ being an excess or defect of the $i-$th component of the background density, triggered by an excitation mechanism. (Note that such trial function for a solitonic excitation has been applied earlier by Dum et al [12].) Inserting the above ansatz into the GP equations (1) and assuming that the excitation mechanism restricts the change of density only to a small interval around $x = 0$ where the TF solutions can be approximated by $\Phi_i(0)$, the following coupled equations can be obtained for the perturbing functions $(i = 1, 2)$

$$\tilde{E}_i\phi_i = \left[-\frac{\hbar^2}{2m_i}\,\partial_{xx} + \sum_{j=1}^{2}\tilde{\Omega}_{ij}\,|\,\phi_j|^2\right]\phi_i \qquad (2)$$

where $\tilde{\Omega}_{ij} \propto \Omega_{ij}$ $(i, j = 1, 2)$ and the very small potential terms have been neglected.

The coupled equations above can be solved for the perturbing functions $\phi_i(x)$ by using the static bright-dark (BD) coupled solitonic ansatz

$$\phi_{B1}(x) = q_1\mathrm{sech}(kx), \qquad \phi_{B1}(x \to \pm\infty) = 0, \qquad (3a)$$
$$\phi_{D2}(x) = q_2\tanh(kx), \qquad \phi_{D2}(x \to \pm\infty) = \pm q_2, \qquad (3b)$$

with the specified boundary conditions, generic complex amplitudes q_i, and common real size parameter k.

The requirement that the moduli of the two amplitudes q_1 and q_2 be positive real numbers gives the stability conditions $(A_{ij} = a_{ij}(1 + m_i/m_j))$

$$f_{B1}(a_{12}) = \frac{A_{12} - A_{22}}{\det(A)} \geq 0, \qquad (4a)$$

$$f_{D2}(a_{12}) = \frac{A_{11} - A_{21}}{\det(A)} \geq 0 \qquad (4b)$$

for the existence of BD solitonic excitation within the two-component BEC. The finite ranges of a_{12} provided by the solitonic stability requirement (4a-b) are naturally narrower than the TF domain of a_{12} prescribed by the positivity of the density $|\,\Phi_i(x)|^2$.

Let us note here that instead of the BD solitonic ansatz (3a-b) one might as well apply BB or DD prescriptions which would still satisfy equations (2). These ansatzes result in similar stability conditions like the forms (4a-b) but yield different stability domain of a_{12}. This is one reason for the usage of the terminology *assessment* by which we mean that the physical value of a_{12} may be well outside the ranges given later in this paper. Nevertheless, for the specific conditions corresponding to the BD stability we shall find compatibility between our findings and others, including the P-test a_{12} values which are obtained from the singularity structure analysis of the coupled GP equations.

From the many possibilities we shall select two examples, with alkali pairs of Li-Na and K-Rb, representing realistic possibilities for future experimental study.

2.1 Static BD solitonic excitations of Li-Na and K-Rb condensates

In order to illustrate the solitonic excitations we show in figure 1 the density profiles $|\tilde{\psi}_i(x)|^2$ of the two BEC systems composed of lithium-sodium and potassium-rubidium

atoms. All of the components are chosen to be in the triplet scattering state. We have kept N_2 fixed by choosing the dark soliton particle number to be $N_2 = 10^4$ for both the ^{7}Li-^{23}Na triplet system and the ^{41}K-^{87}Rb two-component condensate. The bright component has particle number $N_1 \approx 5 \times N_2$ in both cases, corresponding to the chosen values of $a_{12} = 2.9$ nm (Li-Na) and $a_{12} = 4.85$ nm (K-Rb). The excitation of the BEC is manifested by the appearance of the B and D solitons on the TF background density.

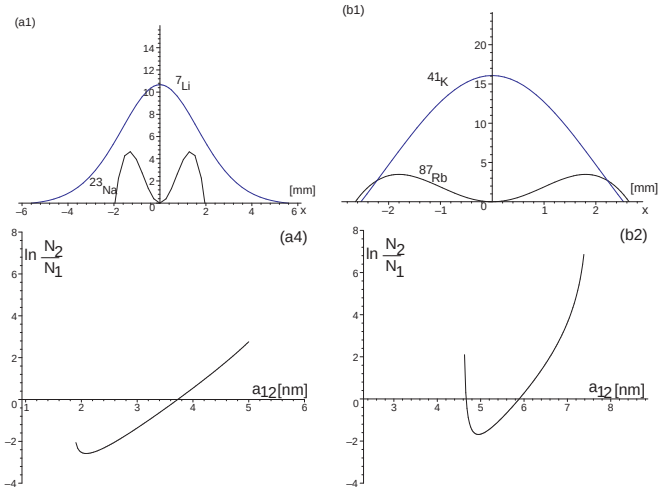

Fig. 1. Absolute squares of wave functions, $|\tilde{\psi_1}|^2$ and $|\tilde{\psi_2}|^2$ in unit $[\mu m^{-1}]$, of the BD solution of the coupled Gross-Pitaevskii equation (1) as a function of the position x for different BEC systems. (a) triplet ^{7}Li-^{23}Na system: $N_2 = 10^4$, $N_1 \approx 4.7 \times 10^4$, $a_{11} = -1.4$ nm (^{7}Li), $a_{22} = 4.0$ nm (^{23}Na) $a_{12} = 2.90$ nm, trap frequencies are: $\omega_1 = 2\pi \times 530$ Hz, $\omega_2 = 2\pi \times 292$ Hz. (b) triplet ^{41}K-^{87}Rb system: $N_2 = 10^4$, $N_1 \approx 4.9 \times 10^4$, $a_{11} = 3.4$ nm (^{41}K), $a_{22} = 5.5$ nm (^{87}Rb), $a_{12} = 4.85$ nm, trap frequencies are: $\omega_1 = 2\pi \times 220$ Hz, $\omega_2 = 2\pi \times 150$ Hz. Figures (a2) and (b2) show the dependence of the logarithmic ratio $\ln(N_2/N_1)$ on the interspecies scattering length a_{12} for these two-component BECs. The transverse crossing area $A = r^2\pi$ with radius $r = 0.3$ nm has been employed throughout.

If the system parameters m_1, m_2, a_{11} and a_{22} are given, then one can determine the broadest range of the interspecies scattering length by using the inequalities (4a-b). This procedure does not involve the particle numbers N_1 and N_2 and gives an assessment of a_{12} range for which the existence of BD or DB solitons in the two-component BEC might be expected. Particle number conservation further restricts the interval of a_{12} and leads to a system coupled equations for N_2/N_1 and the size parameter k (see eqs. (11) of Ref. [1]). Solving these equations, the final results are shown in figures 1(a2) and 1(b2). We observe here a rather smooth behavior of N_2/N_1, as well as a shrinking of the allowable intervals from $a_{12} = -0.6$; 6.1 nm (as determined by inequalities (4a-b)) to $a_{12} = 1.9$; 4.9 nm (obtained by use of particle number conservation) for the ^{7}Li-^{23}Na system, and from $a_{12} = 4.1$; 7.5 nm to $a_{12} = 4.5$; 7.4 nm for the ^{41}K-^{87}Rb configuration.

In conclusion we can say that if the actual value of a_{12} determined by some experimental or theoretical method turns out to fall within the range determined by inequalities (4a-b) then one may fine tune the particle number ratio N_2/N_1 and the size parameter k. In this way we get complete information about both the background and solitonic stability.

2.2 a_{12} values resulting from Painlevé analysis

In Ref. [25] it has been shown that the coupled GP equations pass the Painlevé (P) test provided the following condition among the system parameters (masses, interaction strengths) is satisfied

$$a_{12} = \frac{(m_1 a_{11} + m_2 a_{22})}{M(m_1 + m_2)} \pm \frac{\sqrt{(m_1 a_{11} + m_2 a_{22})^2 + 4M(M-2)m_1 m_2 a_{11} a_{22}}}{M(m_1 + m_2)} \quad (5)$$

with $M \equiv \left[(2m+1)^2 + 7\right]/16$, where m can be treated as an arbitrary positive integer number ($m = 0, 1, 2, \ldots$). This expression depends only on the ratios m_1/m_2, a_{11}/a_{12}, and a_{12}/a_{22} involving the characteristic parameters of the BEC, and the external potential families can be classified by the different values of m. Experimentally preferred spatially harmonic ($\sim x^2$) potential-families fall into the category $m = 2$ ($M = 2$). The condition given by equation (5) can be written in the following interesting form $M = A_{22} f_{D2}(a_{12}) - A_{11} f_{B1}(a_{12})$. This relation connects two independent analysis, namely the solitonic stability conditions (4a-b) and the P test encoded into equation (5). The left hand side is responsible for the fulfillment of the P test which constitutes one of the ingredients of integrability conditions [26] meaning among many others that solitonic solution of the GP equations can be found by the method of inverse scattering transform. Right hand side contains the functions f_{B1} and f_{D2} which should be positive in order that the coupled GP equations may have static solitonic solutions of BD or DB types.

3 Results and discussion

3.1 a_{12} values allowing stability and BD solitonic excitations of BECs

In this subsection the BD stability ranges of a_{12} will be presented in tabulated form, using both triplet and singlet intraspecies data as input, for ninety pairs of alkali atoms. Note that the constraint arising from TF stability is not included in the data given.

Tables 1 and 2 give, separated from each other by semicolons, the limiting point values of the intervals of the interspecies scattering length a_{12} satisfying the stability

conditions encoded into inequalities (4a-b). These conditions express the possibility of developing solitonic excitations of static BD or DB type from the ground state of the two-component BECs, irrespective of the particle numbers N_1 and N_2 of the mixture. The data have been calculated, respectively, with triplet and singlet intraspecies scattering lengths a_{ii} $(i = 1, 2)$ listed in the columns headed by the labels $a_{\rm t}$ and $a_{\rm s}$. The exception is the ^{133}Cs atom where only the average scattering length ($a_{ii} = +\sqrt{\sigma_{exp}/8\pi}$) has been available (therefore the parenthesis).

In general we observe in tables 1 and 2 that one of the limiting points of the intervals of the corresponding DB and BD cases coincides; a fact easily derivable by using the relation $0 = f_{B1}(a_{12}) = f_{D2}(a_{12})$. The diagonal elements of tables 1 and 2 represent one-component BECs which is out of scope of the present interest so that the data are missing there. Furthermore, the rows corresponding to ^{7}Li, ^{39}K, and ^{85}Rb of table 1 do not contain any results as well, if their partner owns a positive triplet intraspecies scattering length. The lack of information means here that we do not find any region of triplet interspecies scattering length a_{12} which gives rise to a stable DB type excitation. On the other hand all items of the column headed by ^{7}Li, ^{39}K, and ^{85}Rb are filled by data. This means that there are definite ranges of triplet a_{12} which support BECs with excitations of BD type where the ^{7}Li, ^{39}K, and ^{85}Rb atoms play the role of the B component (owing to their attractive interaction character). The above argumentation can be confirmed by using stability conditions (4a-b) with $a_{11} < 0$ and $a_{22} > 0$.

In the case when both a_{11} and a_{22} are positive we observe the existence of BECs with both DB or BD formations within a finite interval of positive interspecies scattering length a_{12}. This can be understood physically in the following way. In the context of the mean field theory, an effective potential is created by the i-th atoms acting repulsively among each other (positive scattering lengths) and thus forming the cavity of the D component. Inside this cavity there is a possibility for the j-th atoms to form a bunch representing the B component despite their inherent repulsive character.

The values of a_{12} given in tables 1 and 2 can be used for orientation and in design of two-component BECs. One starts from known intraspecies scattering lengths a_{ii} $(i = 1, 2)$ which are represented by the data denoted by a_t or a_s in the tables. Then one performs the range determination by using inequalities (4a-b). Now one is provided with guessed values of interspecies scattering length a_{12} which can maintain stable two-component BEC configuration. If, from some other sources, we are assured that the actual physical values of a_{12} falls outside the range calculated then one may try either to move the a_{ij} values into the proper direction by utilizing the Feshbach-resonance procedure [22] or to carry out BB or DD stability analysis to learn what type (if any) of static solitonic excitations of the components are possible (suggesting, at the same time, stability of the ground state configurations as well).

However, if all a_{ij}'s $(i, j = 1, 2)$ satisfy inequalities (4a-b), then one may start to carry out the production of two-component BEC using information obtained by fine tuning the component numbers N_i as described in Ref. [1].

3.2 a_{12} from earlier studies

There are very few interspecies scattering length a_{12} data in the literature. The cases studied so far include the pairs composed of different isotopes of rubidium atoms [10], of those of potassium atoms [9], of potassium and rubidium atoms [5, 6], of cesium and rubidium atoms [11], and of lithium and cesium atoms [7]. In the latter case the sign of a_{12} remains undetermined since we can derive, by using the relation $\sigma_{\rm el} = 4\pi a_{12}^2$, only the absolute value of the interspecies scattering length from the measured unpolarized elastic cross-section data $\sigma_{\rm el}(^7{\rm Li}-^{133}{\rm Cs}) \approx 5 \cdot 10^{-12}$ cm^2 to be $a_{12}(^7{\rm Li}-^{133}{\rm Cs}) = \pm 6.3$ nm. This value of a_{12} is to be compared to our ranges of a_{12} characterized by the limiting points $-0.1; 4.6$ and $0.9; 4.6$ nm obtained, respectively, with triplet and singlet intraspecies parameters $a_{11} = -1.4$ and 1.7 nm for the lithium atoms, and $a_{22} = 2.4$ nm for the cesium atom (where the latter value also represents unpolarized data [27] with known positive sign). From the comparison we predict the plus sign for the unpolarized interspecies scattering length, i.e., $a_{12}(^7{\rm Li}-^{133}{\rm Cs}) = +6.3$ nm, provided the stability can be attained.

For the potassium-rubidium system we compare our ranges $a_{12}(^{39}{\rm K}-^{87}{\rm Rb}) = -0.6; 7.6$ nm and $a_{12}(^{41}{\rm K}-^{87}{\rm Rb}) = 4.1; 7.5$ nm obtained using triplet intraspecies data (i.e. with $a_{11}(^{39}{\rm K}) = -0.9$ nm, $a_{11}(^{41}{\rm K}) = 3.4$ nm, and $a_{22}(^{87}{\rm Rb}) = 5.5$ nm) to the triplet interspecies values $a_{12}(^{39}{\rm K}-^{87}{\rm Rb}) = 1.6$ nm [5] and $a_{12}(^{41}{\rm K}-^{87}{\rm Rb}) = 8.1^{+0.5}_{0.3}$ nm given in Ref. [6]. We see here that our findings are comparable with the calculated values of a_{12}.

For the cesium-rubidium system a recent work [11] lists various a_{12} values calculated using six different ab-initio potential for both triplet and singlet collisions. The calculated data lying between $a_{12} = 2; 19$ nm for the singlet and between $a_{12} = -8.5; 3.2$ nm for the triplet ^{133}Cs-^{85}Rb system compare well to ours given by the BD ranges of $a_{12}(^{133}{\rm Cs}-^{85}{\rm Rb}) = 16.9; 97$ nm and $a_{12}(^{85}{\rm Rb}-^{133}{\rm Cs}) = 3.0; 16.9$ nm for the singlet and $a_{12}(^{85}{\rm Rb}-^{133}{\rm Cs}) = -14.8; 2.9$ nm for the triplet arrangement, respectively. In the case of the ^{87}Rb-^{133}Cs DB system the singlet scattering lengths are comparable; for example, the calculated singlet value of $a_{12} = 3.0$ nm of the ab-initio result VI [11] compares well to our range of $a_{12}(^{87}{\rm Rb}-^{133}{\rm Cs}) = 2.9; 3.3$ nm.

Finally, we mention the singlet interspecies scattering length for different isotopes of the potassium atoms, namely the value $a_{12}(^{39}{\rm K}-^{41}{\rm Kb}) = 5.9$ nm [9] to be compared to our range $a_{12}(^{39}{\rm K}-^{41}{\rm K}) = 5.7; 7.0$ nm of the DB case. Again, a clear compatibility is observed.

We may thus conclude that the solitonic excitation method carried out in section 2 is capable to assess approximate intervals of interspecies scattering length a_{12} which may encompass the actual values themselves.

3.3 a_{12} values derived from Painlevé test

In table 3 the values of a_{12} calculated from equation (5) are shown for the harmonic oscillator potential obtained with singlet (upper diagonal) and triplet (lower diagonal) intraspecies scattering lengths a_{ii}, $i = 1, 2$. Although $a_{12} = 0$ is always a solution of equation (5) at $M = 2$, this value is not shown in table 3 because it represents the uncoupled case.

Since the P test of a nonlinear partial differential equation primarily serves to explore its singularity structure, it is only implicitly connected with soliton formation. Therefore we do not in general expect that all values of table 3 match with those encompassed by the a_{12} intervals of table 1 and 2. Yet, the values of items in tables 1 and 3 (lower diagonal) and tables 2 and 3 (upper diagonal) compare well. From this comparison we can learn that only those DB excitations are possible in which the D component is represented by the heavier elements listed in the first line of table 3. Exceptions are the ^{7}Li atom which always plays the role of the B component, and the ^{85}Rb atom which always constitutes the role of the D component (when singlet scattering is considered). This result is physically reasonable, since in a mixture of atoms with different masses and comparable (positive) interspecies scattering lengths, the atoms of the slower (more massive) component is likely to form a dip of D soliton, into which the atoms of the swifter (lighter) component can be captured to create a B soliton. The exceptions occur if the repulsion/attraction between like atoms is strong enough as in the case of ^{85}Rb/^{7}Li to cause a situation with permanent D/B components.

4 Summary

The assessment of the interspecies interaction parameter a_{12} at which two-component atomic BEC exhibits stable configuration is an important task, and can be carried out with the present method in the following way.

In principle, all the ground state parameters including a_{12} can be extracted from the TF equilibrium density profiles. This method is rather ambiguous and, therefore, restricting the number of uncertain parameters or their ranges may prove useful in design of two-component BECs. Moreover, in this fitting procedure the particle numbers are not coupled, they can be chosen independently of each other. The coupling is represented mostly by the inter- and intraspecies scattering length parameters a_{ij} $(i, j = 1, 2)$ being strongly correlated with each other.

If one is interested in producing excitations of BEC in a form of static solitons then one first determines a region of interspecies scattering length a_{12} which may support the soliton formation. This task can be performed by simultaneous solution of the inequalities (4a-b) for a_{12}, provided the interaction parameters a_{11} and a_{22} are given. This step is again free of the particle number parameters N_1 and N_2 and gives the broadest region of a_{12} for which DB or BD static solitons ever can be created inside the stable two-component BEC. For the two examples treated explicitly, table 1 contains such intervals between $a_{12} = -0.6 : 6.1$ nm for the ^{7}Li-^{23}Na triplet system, and $a_{12} = 4.1; 7.5$ nm in the case of the ^{41}K-^{87}Rb triplet scattering.

If the actual value of intraspecies scattering length a_{12} *which must be known from other source*, is within the interval determined above, then one may perform a fine tuning for the particle number ratio N_2/N_1 at which the solitonic excitation may be implemented. This fine tuning is accomplished by solving the coupled nonlinear equations (11) of Reference [1] for the ratio N_2/N_1 and the size parameter k of the solitons. For the two explicit examples chosen, figures 1 contain the results with N_2 kept fixed.

Bright-dark soliton combinations in one- and two-component condensates have already been considered in a variety of papers, including [12, 14, 15, 17]. Experimental creation of both bright [16] and dark [13] solitons have been observed in case of one-component condensates. We hope that fine tuning the system parameters (scattering lengths, particle numbers, trapping frequencies) as outlined above may also help in realizing coupled solitonic excitations in two-component Bose-Einstein condensates.

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Table 2. Intervals of singlet interspecies scattering lengths a_{12} assessed by static solitonic excitations of two component BECs composed of alkali pairs indicated. The column a_S gives the singlet scattering length a_{ii} of atoms taken from the references: ^{7}Li [29, 32, 33, 41], ^{23}Na [34, 35], ^{39}K, ^{41}K [9], ^{83}Rb, ^{85}Rb, ^{87}Rb [10], ^{133}Cs, ^{135}Cs [27, 39]. (For ^{1}H the triplet value has been used [28]). All values are in unit of nm.

			Bright-component									
	Element	a_S	^{1}H	^{7}Li	^{23}Na	^{39}K	^{41}K	^{83}Rb	^{85}Rb	^{87}Rb	^{133}Cs	^{135}Cs
Dark-component	**^{1}H**	(0.1)		-0.0;0.2	0.0;0.2	0.0;0.2	0.0;0.2	0.0;0.1	0.0;0.6	0.0;0.1	0.0;0.1	0.0;0.2
	^{7}Li	1.7	0.2;3.0		0.8;1.8	0.5;2.5	0.5;1.6	0.3;1.3	0.3;7.7	0.3;1.5	0.2;0.9	0.2;2.9
	^{23}Na	2.7	0.2;5.3	1.8;4.2		2.0;4.3	1.9;3.3	1.2;2.5	1.2;15.1	1.1;2.9	0.8;1.8	0.8;6.0
	^{39}K	7.3	0.2;14.2	2.5;12.4	4.3;9.2		5.7;7.0	4.6;4.7	4.6;28.0	4.5;5.4	3.3;3.5	3.3;11.5
	^{41}K	4.4	0.2;8.6	1.2;5.4	3.3;5.7	4.6;5.7		3.0;3.7	2.9;22.1	2.9;4.3	2.1;2.8	2.0;9.0
	^{83}Rb	3.4	0.1;6.8	1.3;6.3	2.5;5.4	4.6;4.7	3.7;4.6		3.4;20.7	3.3;4.0	2.6;2.8	2.6;9.2
	^{85}Rb	124.8	0.6;246	7.7;230	15.1;196	28.0;171	22.1;167	20.7;126		24.2;123	16.9;97	55.5;96
	^{87}Rb	4.7	0.1;9.2	1.5;8.7	2.9;7.4	5.4;6.5	4.3;6.3	4.0;4.8	4.7;24.2		3.3;3.8	3.7;10.8
	^{133}Cs	(2.4)	0.1;4.8	0.9;4.6	1.8;4.0	3.5;3.7	2.8;3.7	2.8;3.0	3.0;16.9	2.9;3.3		2.7;7.9
	^{135}Cs	26.0	0.2;51.6	2.9;49.5	6.0;44.4	11.5;40.3	12.1;39.9	9.1;32.3	32.0;55.5	10.8;31.6	7.9;26.2	

Table 1. Intervals of triplet interspecies scattering lengths a_{12} assessed by static solitonic excitations of two component BECs composed of alkalis indicated. The column a_t gives the triplet intraspecies scattering length a_{ii} taken from the references: ^{1}H [28] , ^{7}Li [29–33], ^{23}Na [34–36], ^{39}K, ^{41}K [9, 37], ^{83}Rb, ^{85}Rb, ^{87}Rb [10, 38], ^{133}Cs, ^{135}Cs [27, 39, 40]. All values are in unit of nm.

			Bright-component									
	Element	a_t	^{1}H	^{7}Li	^{23}Na	^{39}K	^{41}K	^{83}Rb	^{85}Rb	^{87}Rb	^{133}Cs	^{135}Cs
Dark-component	**^{1}H**	0.1		-2.4;0.0	0.0;0.2	-1.8;0.0	0.0;0.2	0.0;0.1	-37.6;0.0	0.0;0.2	0.0;0.1	0.0;0.1
	^{7}Li	−1.4	–;–		–;–	-1.5;-0.8	–;–	–;–	-35.1;-2.7	–;–	–;–	–;–
	^{23}Na	4.0	0.2;7.7	-0.6;6.1		-1.1;3.0	2.8;3.5	1.7;3.4	-30;1.7	1.7;3.8	1.2;2.2	1.2;3.8
	^{39}K	−0.9	-0.4;-0.1	–;–	–;–		–;–	–;–	-26;-3.8	–;–	–;–	–;–
	^{41}K	3.4	0.1;6.6	-0.4;5.8	3.5;4.4	-0.9;3.5		2.3;3.6	-25.4;2.2	2.2;4.0	1.6;2.4	1.6;4.2
	^{83}Rb	4.2	0.1;8.3	-0.2;7.7	3.4;6.6	-0.6;5.7	3.6;5.6		-19.2;4.1	4.1;4.8	3.1;3.2	3.2;5.3
	^{85}Rb	−19.0	–;–	-2.7;-0.2	–;–	-3.8;-0.6	–;–	–;–		–;–	–;–	–;–
	^{87}Rb	5.5	0.2;10.9	-0.2;10.2	3.8;8.7	-0.6;7.6	4.1;7.5	4.8;5.6	-18.8;5.6		3.6;4.4	4.3;6.1
	^{133}Cs	(2.4)	0.0;4.8	-0.1;4.6	2.2;4.1	-0.4;3.7	2.4;3.7	3.0;3.1	-14.8;2.9	2.9;3.6		2.4;4.2
	^{135}Cs	7.2	0.1;14.3	-0.1;13.7	3.8;12.3	-0.4;11.2	4.2;11.1	5.3;8.9	-14.7;8.9	6.1;8.8	4.2;7.3	

Table 3. Values of interspecies scattering length a_{12} solving the P test condition, equation (5), at classification numbers $m = 2$ preferring harmonic oscillator trapping potentials. Upper diagonal contains results obtained with singlet intraspecies scattering lengths $a_{\rm S}$ listed in the second column, lower diagonal exhibits those with triplet a_{ii}'s given in the third column as $a_{\rm t}$. All values are in unit of nm.

Element	$a_{\rm S}$	$a_{\rm t}$	^{1}**H**	^{7}**Li**	^{23}**Na**	^{39}**K**	^{41}**K**	^{83}**Rb**	^{85}**Rb**	^{87}**Rb**	^{133}**Cs**	^{135}**Cs**
^{1}**H**	(0.1)	0.1		-1.3	2.6	7.1	4.3	3.4	123.3	4.6	2.4	25.8
^{7}**Li**	1.7	−1.4	-1.2		2.5	6.4	2.9	3.3	115.6	4.5	2.3	24.8
^{23}**Na**	2.75	4.0	3.8	2.9		5.6	3.8	3.3	196.7	4.3	2.5	22.6
^{39}**K**	7.28	−0.9	-0.9	-0.9	0.9		5.8	4.7	87.9	5.5	3.5	21.8
^{41}**K**	3.12	3.4	3.3	2.8	3.6	1.3		3.8	85.0	4.6	2.6	20.6
^{83}**Rb**	3.43	4.2	4.2	3.8	4.1	2.6	3.9		64.8	4.1	2.8	17.4
^{85}**Rb**	124.8	−19.0	-18.8	-17.6	-14.1	-13.3	-11.6	-7.5		64.0	50.3	64.3
^{87}**Rb**	4.68	5.5	5.4	5.0	5.2	3.5	4.8	4.8	-6.6		3.3	17.6
^{133}**Cs**	(2.4)	(2.4)	2.4	2.2	2.6	1.6	2.6	3.1	-6.0	3.6		4.8
^{135}**Cs**	26.0	7.2	0.7	6.8	6.7	5.4	6.3	6.1	-2.9	6.5	4.8	